AN ANALYTICAL MODEL FOR NEAR-FAULT GROUND MOTIONS AND THE RESPONSE OF SDOF SYSTEMS

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ABSTRACT

An analytical model for fault-normal near-fault ground motions, which can be used in lieu of recorded ground motions, is described. The ground motion model is defined by five parameters that, for a recorded near-fault ground motion, can be determined by a nonlinear regression analysis.

Time-history results for linear and nonlinear single-degree-of-freedom systems are used to demonstrate the ability and limitations of the proposed model to predict the severity of the structural response caused by near-fault ground motions. For ductility ratios commonly encountered in practice, the proposed ground motion model predicts displacement demands that, on average, are within 10% of those caused by the recorded ground motions for systems that have an initial period, T, that lies in the range $0.65 \leq T/T_p \leq 1.5$, where T_p is the period of the velocity pulse present in the record. For systems that have periods that lie outside of this range, the proposed ground motion model tends to underestimate the displacement demands because it cannot replicate the frequency content of the recorded ground motions beyond that associated with the velocity pulse.

Introduction

The manner in which a structure sustains damage during an earthquake is strongly influenced by its proximity to the rupturing fault. For structures located within 15 km of the fault, the damage is often incurred during one or two cycles of severe inelastic deformations that coincide with a large amplitude velocity pulse in the fault-normal component of the ground motion. In light of this observation, an analytical model for the velocity pulse, which can be used in lieu of recorded ground motions, is proposed. Similar near-fault ground motion models have been suggested by other researchers. In particular, Alavi and Krawinkler (2000) consider three piecewise-linear equivalent velocity pulses and investigate their suitability for seismic performance assessments. Likewise, Makris and Chang (2000) describe several families of cycloidal pulses that can be used to represent near-fault ground displacements and examine the transient behavior of single-degree-of-freedom oscillators with viscous and friction damping when they are subjected to these idealized ground motions. The ground motion model described

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in this paper serves as an alternative to these existing models. As we will see, the proposed model is defined by five parameters that, for a specified near-fault record, are determined by a nonlinear regression analysis.

We note that a time-history analysis conducted with the proposed ground motion model will not capture all the details of the structural response. However, this is not the objective of such analyses. The intent of time-history analyses using the proposed model is to predict the extreme values of those responses that correlate well with the seismic performance of a structure. In this paper, we present time-history results for linear and nonlinear single-degree-of-freedom (SDOF) systems that demonstrate the ability and limitations of the proposed model to predict the severity of the structural response caused by near-fault ground motions.

Near-fault Ground Motion Model

Consider a near-fault ground motion model in which the fault-normal component of the ground velocity is idealized as

$$\begin{aligned} \dot{u}_m(t;\boldsymbol{\theta}) &= V_p \exp\left[-n_1\left(\frac{3}{4}T_p - t + t_0\right)\right] \sin\left[\frac{2\pi}{T_p}(t-t_0)\right] && t_0 < t \le t_0 + \frac{3}{4}T_p \\ &= V_p \exp\left[-n_2\left(t - t_0 - \frac{3}{4}T_p\right)\right] \sin\left[\frac{2\pi}{T_p}(t-t_0)\right] && t_0 + \frac{3}{4}T_p < t \le t_0 + 2T_p \qquad (1) \\ &= 0 && \text{otherwise,} \end{aligned}$$

where $\boldsymbol{\theta} = [V_p, T_p, t_0, n_1, n_2]^T$ is the vector of parameters that define the model. In particular, V_p and T_p characterize the amplitude and period of the velocity pulse, respectively, t_0 defines the time at which the pulse starts and n_1 and n_2 are shape parameters. Typical fault-normal ground velocities generated by this model are plotted in Fig. 1, which demonstrates the variety of pulse types that can be obtained by varying the shape parameters n_1 and n_2. Also, note the secondary role played by parameter t_0, which only serves to locate the pulse along the time axis. However, we will see shortly that t_0 plays an important role when fitting the model defined by Eq. 1 to a recorded near-fault ground motion.

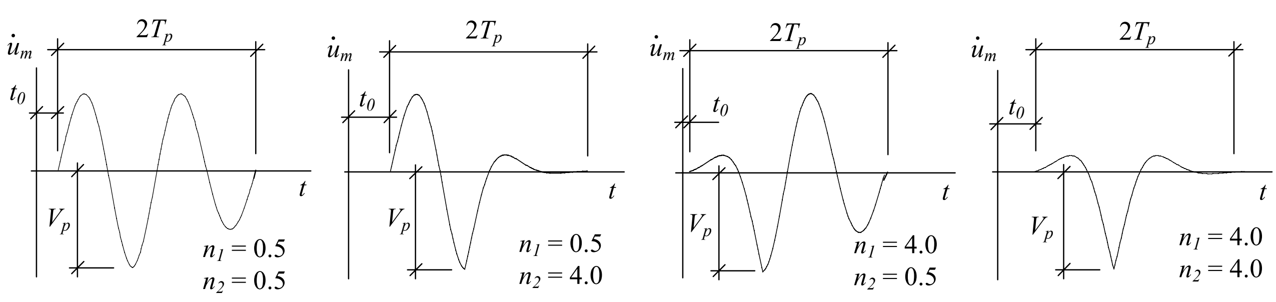

Figure 1. Typical fault-normal velocity pulses generated by proposed model defined by Eq. 1.

The parameter values $\boldsymbol{\theta}^*$ that define the ground velocity model $\dot{u}_m(t;\boldsymbol{\theta}^*)$ that best fits a recorded fault-normal near-fault ground velocity $\dot{u}_g(t)$ can be determined by a nonlinear regression; in particular, $\boldsymbol{\theta}^*$ is that realization of $\boldsymbol{\theta}$ that minimizes the sum of squares

$$S(\boldsymbol{\theta}) = \sum_{i=1}^{N} \left[\dot{u}_g(t_i) - \dot{u}_m(t_i;\boldsymbol{\theta})\right]^2, \tag{2}$$

where t_i, $i = 1, 2, \ldots, N$ are the times at which $\dot{u}_g(t)$ has been recorded. Newton's method is commonly employed to determine $\boldsymbol{\theta}^*$ (Seber and Wild, 1989). Unfortunately, for the case at hand, the solution obtained by Newton's method is strongly dependent upon the initial values assigned to the parameters t_0 and T_p, i.e., the assumed location and duration of the pulse. When poorly chosen initial values of t_0 and T_p are used, Newton's method often converges to a local minimum of $S(\boldsymbol{\theta})$, rather than the desired global minimum. To circumvent this problem in an efficient manner, the minimization of (2) is carried out in two steps. First, an approximate solution is determined by means of a genetic algorithm (Coley, 1999), which is a form of importance sampling that randomly searches the parameter space for candidate solutions and then refines and concentrates its search around those solutions that result in the smallest values of $S(\boldsymbol{\theta})$. The genetic algorithm is terminated when it is unable to improve the solution after a prescribed number of iterations. The approximate solution found by the genetic algorithm is then used as the initial value for Newton's method, which is used to refine the solution. We note however that there is no guarantee that the above approach will locate the global minimum. This is a well-known shortcoming of all nonlinear minimization routines. To address this problem, the genetic algorithm is restarted several times to obtain a number of approximate solutions. Each of these solutions is then used as the starting point for Newton's method. Our experience with this approach has revealed that when the genetic algorithm is restarted five times, usually four or five of the solutions lead to the same global minimum of $S(\boldsymbol{\theta})$ after applying Newton's method.

Recorded and Fitted Ground Motions

Using the procedure described in the previous section, the ground velocity model defined by Eq. 1 is fit to an ensemble of ten recorded fault-normal near-fault ground velocities originally compiled by Somerville et al. (1997) for Phase II of the SAC[3] Joint Venture Steel Project. The ground motions and the fitted model parameters are summarized in Table 1. Also listed in this table is the peak ground velocity, $\dot{u}_{g\,\max}$, and predominant period, T_{Sv} (defined as the period at which the pseudo-velocity response spectrum assumes its maximum value), of each recorded ground motion.

As illustrated in Figs. 2 and 3, the model parameters V_p and T_p appear to be related to $\dot{u}_{g\,\max}$ and T_{Sv}, respectively. Alavi and Krawinkler (2000) noted similar relationships for the amplitude and period parameters in their equivalent pulses. Based on the roles played by V_p and T_p in Eq. 1, the relationships seen in Figs. 2 and 3 are sensible and give us reason to believe that robust attenuation relationships can be developed for V_p and T_p. These relationships may be in terms of the expected values of $\dot{u}_{g\,\max}$ and T_{Sv} at a building site or possibly in terms of more

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Table 1. Recorded ground motions and fitted ground motion model parameters.

Ground Motion	M_w	R (km)	$\dot{u}_{g\,\max}$ (cm/s)	T_{Sv} (s)	V_p (cm/s)	T_p (s)	n_1	n_2	t_0 (s)
1. Tabas, Iran (1978)	7.4	1.2	128	4.50	98	5.26	0.16	0.14	8.34
2. Loma Prieta, Los Gatos (1989)	6.9	3.5	173	3.25	126	3.17	0.29	0.33	5.89
3. Loma Prieta, Lex. Dam (1989)	6.9	6.3	179	1.07	191	2.55	0.94	10.47	2.43
4. Cape Mendocino (1992)	7.1	8.5	149	0.74	160	1.75	0.75	11.78	1.96
5. Erzincan, Turkey (1992)	6.7	2.0	119	2.30	114	2.41	2.59	0.77	0.95
6. Landers (1992)	7.3	1.1	132	4.41	122	8.23	1.03	0.43	3.58
7. Northridge, Rinaldi (1994)	6.7	7.1	174	1.05	197	1.18	4.68	1.88	1.47
8. Northridge, Sylmar (1994)	6.7	6.4	122	2.40	108	2.72	8.88	0.44	1.59
9. Kobe, JKMA (1995)	6.9	0.6	160	0.88	93	0.90	0.22	-0.66	7.12
10. Kobe, Takatori (1995)	6.9	4.3	174	1.27	156	2.05	0.67	0.61	3.45

fundamental measures of the site's seismic environment such as the magnitude, M_w, and distance, R, of the anticipated earthquake.

The recorded ground velocities and fitted models for representative ground motions listed in Table 1 are plotted in Figs. 4, 5, 6 and 7. Also shown in these figures are the corresponding displacements and accelerations obtained by integrating and differentiating the velocity records, respectively. It is evident from Figs. 4 through 7 that the proposed ground motion model can replicate the velocity pulse present in these records; however, the model appears to be unable to properly represent the frequency content of the ground motions beyond that associated with the velocity pulse. The implications of these observations are investigated below by subjecting SDOF systems to each of the recorded ground motions and corresponding fitted ground motion models listed in Table 1 and comparing the responses.

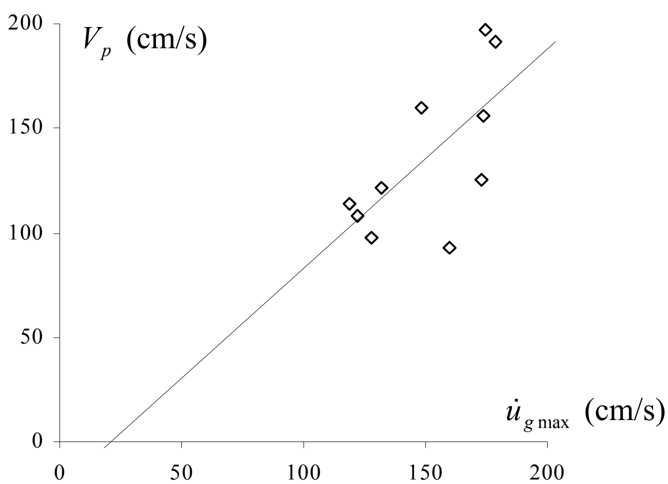

Figure 2. Ground motion model amplitude, V_p, vs. peak ground acceleration of recorded ground motion, $\dot{u}_{g\,\max}$.

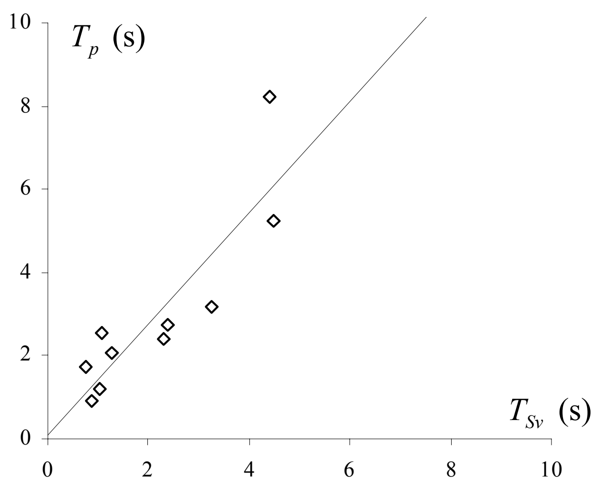

Figure 3. Ground motion pulse period, T_p, vs. predominant period of recorded ground motion, T_{Sv}.

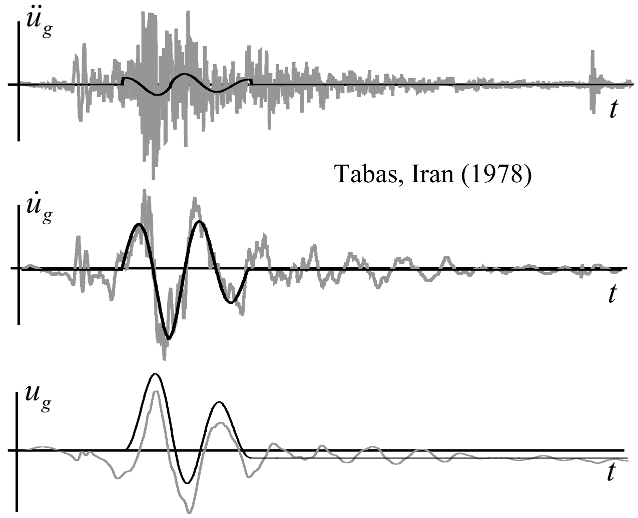

Figure 4. Model fitted to record 1 in Table 1.

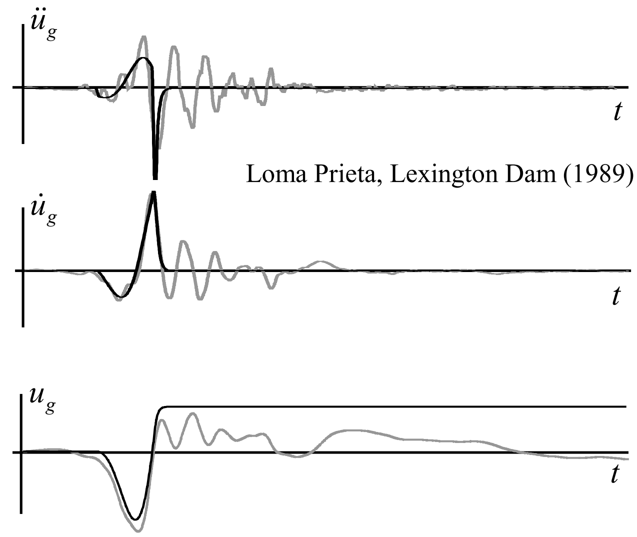

Figure 5. Model fitted to record 3 in Table 1.

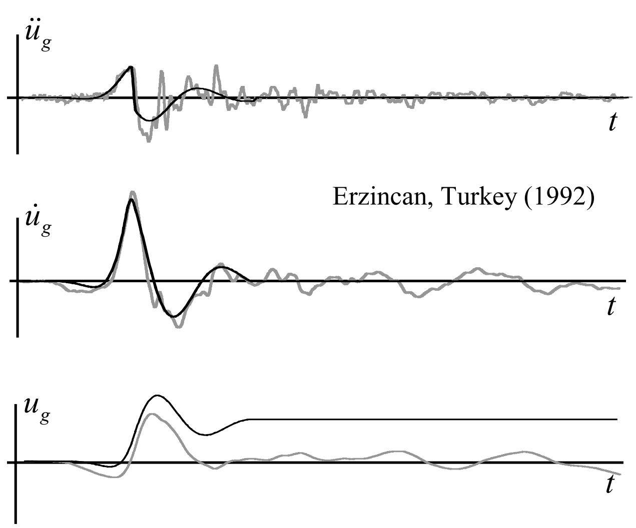

Figure 6. Model fitted to record 5 in Table 1.

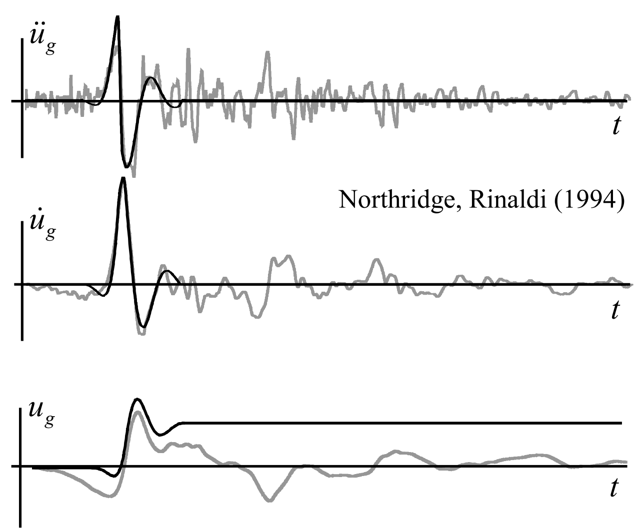

Figure 7. Model fitted to record 7 in Table 1.

SDOF Response to Recorded and Fitted Ground Motions

To assess the suitability of the proposed ground motion model for structural analyses, we consider the response of the SDOF system shown in Fig. 8. The oscillator has initial stiffness k_0, yield strength f_y, post-yield stiffness αk_0, damping ratio ς and mass m. In the following analyses, we set $\varsigma = 0.03$ and $\alpha = 0.10$. The ductility ratio of the system is defined as $\mu = \delta_{\max} / \delta_y$, where $\delta_{\max}$ is the maximum absolute displacement of the oscillator when it is excited by a ground motion and $\delta_y = f_y / k_0$ is the displacement at which the oscillator first yields. For a linear system, $\mu = 1$.

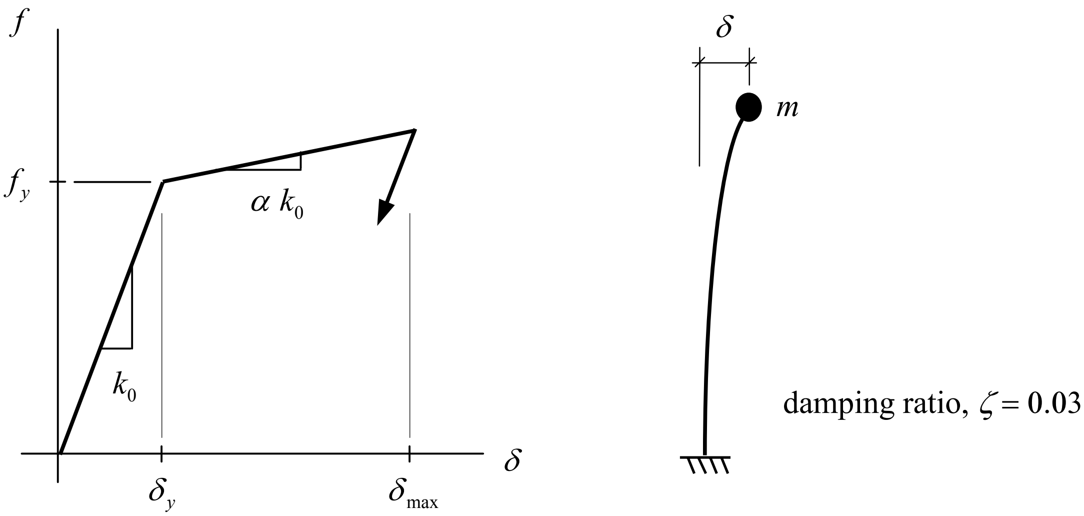

Figure 8. Nonlinear SDOF system considered in analyses.

Constant Ductility Response Spectra

For an oscillator that has initial natural period $T = 2\pi\sqrt{m/k_0}$, let $\delta_{yg}(T,\mu)$ denote the required yield displacement of the system such that the ductility ratio is μ when the oscillator is subjected to recorded ground motion $\ddot{u}_g(t) = d\dot{u}_g(t)/dt$. Similarly, let $\delta_{ym}(T,\mu)$ denote the required yield displacement when the oscillator is subjected to the corresponding model of the ground motion, $\ddot{u}_m(t;\boldsymbol{\theta}) = d\dot{u}_m(t;\boldsymbol{\theta})/dt$. A plot of $\delta_{yg}(T,\mu)$ or $\delta_{ym}(T,\mu)$ as a function of T for a prescribed value of μ is known as a constant ductility response spectrum. Note that, because $\delta_{\max} = \mu\delta_y$, the constant ductility response spectrum can be used to summarize the displacement demands imposed on a nonlinear SDOF system by a ground motion for a range of periods.

For each recorded and fitted ground motion listed in Table 1, $\delta_{yg}(T,\mu)$ and $\delta_{ym}(T,\mu)$ are computed for $\mu = 1, 2, 4, 8$ and $0.05 \le T/T_p \le 5.0$, where T_p is the fitted pulse period listed in Table 1 for that ground motion. The sample mean and mean-plus-or-minus-one-standard-deviation constant ductility response spectra computed for the ensemble of recorded ground motions and for the ensemble of fitted ground motions are plotted in Fig. 9. It is evident in this figure that the constant ductility response spectra computed for the model ground motions are comparable to those computed for the recorded ground motions for periods near $T/T_p = 1$. However, as T/T_p is decreased or increased away from unity, the agreement between the response spectra of the two ensembles weakens. In fact, for $T/T_p < 0.5$, the difference between the mean response spectra is approximately one standard deviation for all ductility ratios; clearly, the proposed model does not perform well in this period range. In contrast, for $T/T_p > 0.75$, the difference between the mean response spectra is a fraction of the record-to-record (aleatory) variability present in the ensembles. However, the proposed ground motion model appears to predict displacement demands that, on average, are smaller than those caused by the recorded ground motions. We can attribute this shortcoming of the proposed ground motion model to the fact that the model only attempts to replicate the velocity pulse. Consequently, only those frequencies near that of the velocity pulse, i.e., near $2\pi/T_p$, are properly represented by the fitted ground motions.

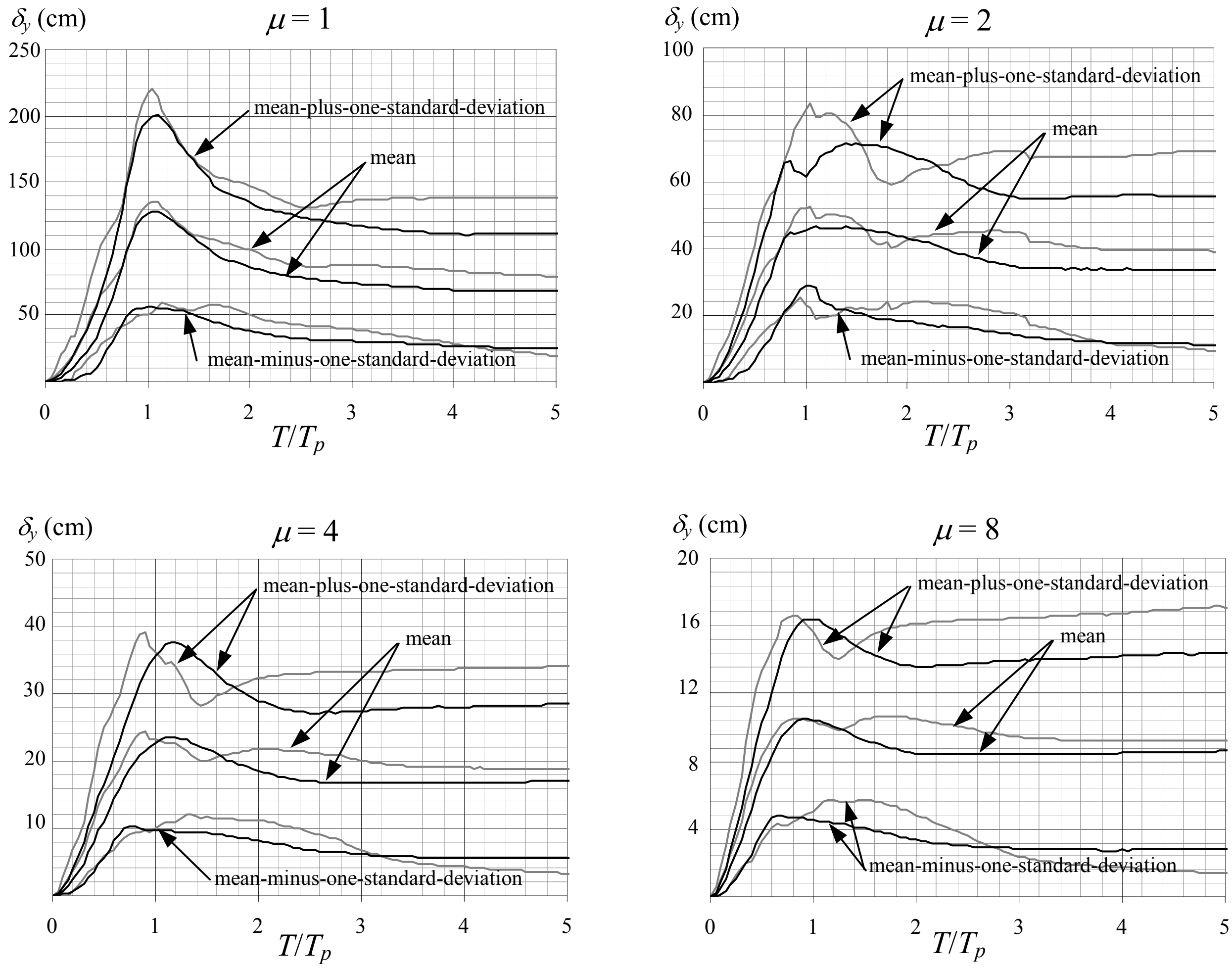

Figure 9. Mean and mean-plus-or-minus-one-standard-deviation constant-ductility response spectra for $1 \leq \mu \leq 8$. Recorded ground motions in gray, fitted ground motions in black.

Response Ratio

To further assess the accuracy of the proposed ground motion model, it is useful to examine the response ratio $\psi(T,\mu) = \delta_{ym}(T,\mu) / \delta_{yg}(T,\mu)$ for each ground motion. It should be apparent that when $\psi(T,\mu) = 1$ the ground motion model predicts displacement demands that are equal to those caused by the recorded ground motion. The sample mean and mean-plus-or-minus-one-standard-deviation curves for $\psi(T,\mu)$ are plotted in Fig. 10, in which it can be seen that the sample mean of the response ratio, $\overline{\psi}(T,\mu)$, is close to unity for some intervals of T/T_p that appear to depend mildly on the ductility ratio. Assuming that $0.9 \leq \overline{\psi}(T,\mu) \leq 1.1$ is sufficiently accurate for engineering calculations, the ranges of T/T_p for which the proposed ground motion model is applicable vary from $0.65 \leq T/T_p \leq 2.3$ for $\mu = 2$ to $0.5 \leq T/T_p \leq 1.5$ for $\mu = 8$. Considering all four ductility ratios, we conclude that the proposed ground motion model predicts displacement demands that, on average, are within 10% of those caused by the recorded ground motions for $0.65 \leq T/T_p \leq 1.5$. For systems that have periods that lie outside of this range, the proposed ground motion model tends to underestimate the displacement demands.

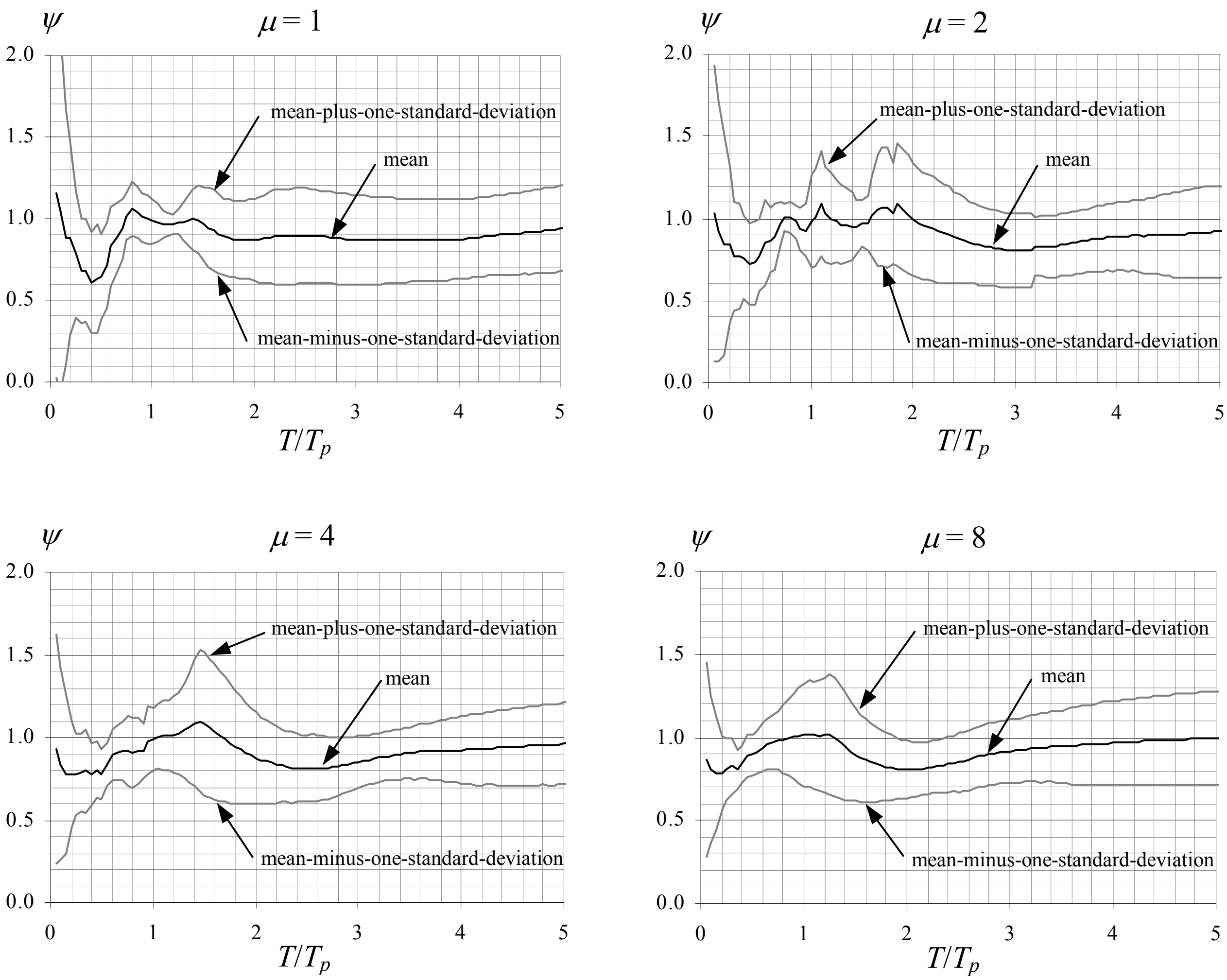

Figure 10. Mean and mean-plus-or-minus-one-standard-deviation response ratio curves for $1 \le \mu \le 8$.

We note that the variability in $\psi(T,\mu)$, which is indicated by the distance between the mean-plus-or-minus-one-standard-deviation curves plotted in Fig. 10, can be partially attributed to model uncertainty, which arises when an imperfect mathematical model (Eq. 1 in this case) is used to a represent complex phenomenon. To see this, consider the ideal situation in which the proposed ground motion model always predicts the displacement demands exactly. For this case, in which all model uncertainty has been eliminated, $\psi(T,\mu)=1$ for all records and the mean-plus-or-minus-one-standard-deviation curves plotted in Fig. 10 would lie on the mean curve. Of course, record-to-record variability would still exist, but it would only be apparent in plots of absolute quantities, such as the constant ductility response spectra plotted in Fig. 9. The same would be true if the model consistently under- or overestimates the maximum displacement, i.e., if $\psi(T,\mu)=\gamma$, where $\gamma \neq 1$ but is the same for all ground motions. Thus, the variability in $\psi(T,\mu)$ apparent in Fig. 10 reveals that for some ground motions the proposed model overestimates the displacement demands, while for others, it underestimates the displacement demands. Contrary to what might be concluded from Fig. 9 alone, the proposed ground motion model does not always underestimate the displacement demands imposed on a structure.

Applications

In light of the results presented in Figs. 9 and 10, it appears that the proposed ground motion model can represent fault-normal near-fault ground motions with sufficient accuracy for structural analyses, provided the initial natural period of the structure is comparable to the period of the velocity pulse. The advantages of using the proposed ground motion model in lieu of recorded ground motions lie in the fact that the model is defined in terms of only four parameters (t_0 is only of interest when fitting the model to a recorded ground velocity) that can be determined from attenuation relationships or simulated from probability distributions that can be readily derived from a constantly-growing database of recorded ground motions. As a result, engineers can use the model to consider realizations of near-fault ground motions beyond those available in the historical record.

One application of the proposed model is to simulate near-fault ground motions for a specified building site. Given the seismic environment of the site, e.g., the distance to active faults and the anticipated magnitudes of earthquakes on those faults, one can use attenuation relationships to estimate sets of parameters to be used in ground motion model. In this way, an ensemble of simulated ground motions can be generated and used, for example, to estimate upper and lower bounds for the response quantities under consideration. Alternatively, the model parameters may be perturbed in a systematic manner to examine the sensitivity of the structural response to the amplitude, duration and shape of the velocity pulse.

The proposed ground motion model is also well suited for structural-reliability-based analyses, in which the performance of a structure is quantified in terms of the probability that it fails to satisfy prescribed limit states. In earthquake engineering applications of structural reliability, it is critical that the intensity and variability of the ground motions are accurately represented. One way of representing the ground motions is with a stochastic model. Unfortunately, stochastic models of near-fault ground motions can be complex and cumbersome to implement due to the strong nonstationary features of such motions. However, the need for a stochastic representation of near-fault ground motions in structural reliability analyses can be avoided by treating the parameters of the proposed ground motion model as random variables. By replacing the stochastic model with a probabilistic model (defined by the joint probability distribution of the four model parameters/random variables), the size and complexity of the structural reliability problem to be solved is reduced while maintaining a sufficiently accurate representation of the intensity and variability of the ground motions.

Summary

A mathematical model for fault-normal, near-fault ground motions, which can be used in lieu of recorded ground motions, is proposed. The ground motion model is defined by five parameters that, for a specified near-fault record, can be determined by a nonlinear regression analysis. The suitability of the proposed ground motion model for structural analyses is evaluated by comparing the response of linear and nonlinear SDOF systems that are subjected to an ensemble of recorded near-fault ground motions and their idealizations, which are obtained by fitting the proposed model to the recorded motions. For ductility ratios $\mu \leq 8$, the proposed ground motion model predicts displacement demands in SDOF systems that, on average, are within 10% of those caused by the recorded ground motions for systems that have an initial period, T, that lies in the range $0.65 \le T / T_p \le 1.5$, where T_p is the period of the velocity pulse present in the record. For systems that have periods that lie outside of this range, the proposed ground motion model tends to underestimate the displacement demands, due to its inability to reproduce the frequency content of the recorded ground motions beyond that associated with the velocity pulse.

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